

科目	工程數學	適用系所	資訊與通訊產業碩士專班	時間	一〇〇分鐘
----	------	------	-------------	----	-------

※請務必在答案卷作答區內作答。

共1頁第1頁

- Find the general solution of $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 3\sin x$ (15%)
- Find the general solution of $\frac{dy}{dx} = -e^{-x}y^2 + y + e^x$ (15%)
- Using Laplace's transform to solve $\frac{d^2y}{dt^2} + 2t\frac{dy}{dt} - 4y = 1$ and $y(0)=y'(0)=0$ (20%)

- Consider the following wave-equation (15%×2=30%)

$$\frac{\partial^2 u(x,t)}{\partial t^2} = 25 \frac{\partial^2 u(x,t)}{\partial x^2}$$

- Solve the equation by using the **Laplace** transform with respect to t

With initial conditions: $u(x,0) = u_t(x,0) = 0, 0 < x < \infty, t > 0$

Boundary conditions: $u(0,t) = e^{-2t} + \int_0^t \sin(t-\tau)u(0,\tau)d\tau, t \geq 0$, as $x \rightarrow \infty$ $u(x,t)$ is bounded.

- Solve the equation by using the **Fourier** transform in x

$$u(x,0) = 2xe^{-2x}H(x), u_t(x,0) = 0, -\infty < x < \infty$$

- Consider the following periodic function $f(x)$ (20%)

$$f(x) = \begin{cases} 9-x^2 & \text{for } -3 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}, f(x) = f(x+6)$$

- Find the Fourier series expansion of $f(x)$ (15%)

- By using the result in (1), to evaluate (5%)

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} + \cdots = ?$$